Part II Paper - III

Monotonic Sequences :
A sequence $\{S_n\}$ is said to be monotonic increasing, if $S_{n+1} \geq S_n$ $\forall$ n and monotonic decreasing if $S_{n+1} \leq S_n$ $\forall$ n. It is said to be monotonic if it is either monotonic increasing or monotonic decreasing

A sequence $\{S_n\}$ is strictly increasing if $\forall$ n, $S_{n+1} > S_n$ and strictly decreasing if $S_{n+1} < S_n$.
The importance of monotonic sequences lies in the fact that they can't oscillate. They either converge or diverge

Th A necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.

N.C The condition is necessary for we know that every convergent sequence is bdd.

S.C The condition is sufficient - Let a bounded sequence $\{S_n\}$ be monotonic increasing. Let S denotes its range, which is evidently bounded. By completeness property S has the least upper bound (the supremum), say M.

We shall show that $\{S_n\}$ converges to M.

Let ϵ be any pre assigned positive number.

Now since $M-\epsilon$ is a number less than the supremum M, there exists at least one member say S_m such that $S_m > M-\epsilon$

As $\{S_n\}$ is a monotonic increasing sequence.

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$S_n \geq S_m > M - \epsilon \ \forall n \geq m$

Again, Since M is the Supremum

$S_n \leq M < M + \epsilon, \ \forall n$

Thus $M - \epsilon < S_n < M + \epsilon, \forall n \geq m$

$\Rightarrow |S_n - M| < \epsilon \ \forall \ n \geq m$

$\Rightarrow \{S_n\}$ converges and $\lim S_n = M$

Corollary 1 – A monotonic increasing bounded above sequence converge to its least upper bound and a monotonic decreasing bounded below to the Greatest lower bound

Corollary 2 Every monotonic increasing sequence which is not bounded above, diverge to $+\infty$

Let $\{S_n\}$ be a monotonic increasing sequence, not bounded above.

Let G be any real number however large.

Hence the sequence $\{S_n\}$ is unbounded above the monotonic increasing

$\exists$ a positive integer m such that

$S_m > G$ and $S_n \geq S_m$, $\forall\, n \geq m$

$\Rightarrow S_n > G$, $\forall\, n \geq m$

Hence $\lim S_n = +\infty$

Corollary 3. Every monotonic decreasing sequence which is not bounded below, diverges to $-\infty$